8th Benelux Mathematical Olympiad

Soest, 29 April – 1 May 2016

Problem 1. Find the greatest positive integer N with the following property: there exist integers $x_1, \ldots, x_N$ such that $x_i^2 - x_i x_j$ is not divisible by 1111 for any $i \neq j$.

Problem 2. Let n be a positive integer. Suppose that its positive divisors can be partitioned into pairs (i.e. can be split in groups of two) in such a way that the sum of each pair is a prime number. Prove that these prime numbers are distinct and that none of these are a divisor of n.

Problem 3. Find all functions $f\colon \mathbb{R} \to \mathbb{Z}$ such that

$$\Big(f\big(f(y) - x\big)\Big)^2 + f(x)^2 + f(y)^2 \;=\; f(y) \cdot \Big(1 + 2f\big(f(y)\big)\Big)$$

for all $x, y \in \mathbb{R}$.

Problem 4. A circle ω passes through the two vertices B and C of a triangle ABC. Furthermore, ω intersects segment AC in $D \neq C$ and segment AB in $E \neq B$. On the ray from B through D lies a point K such that $|BK| = |AC|$, and on the ray from C through E lies a point L such that $|CL| = |AB|$. Show that the circumcentre O of triangle AKL lies on ω.

Language: English

Time available: 4.5 hours

Each problem is worth 7 points